1 stepped pressure equilibrium code : li00aa

Contents

1.1 outline

1. Lagrangian integration: locate high-order periodic field lines that approximate cantori.
2. Cantori form important barriers that can severely restrict field line transport and thus anisotropic heat transport [1].

1.1.1 Lagrangian integration

1. Magnetic field lines are curves that extremize the action integral [2],

$$S \equiv \int_{\mathcal{C}} \mathbf{A} \cdot d\mathbf{l}, \tag{1}$$

where $\mathbf{A} = A_\theta \nabla\theta + A_z \nabla\zeta$ is the magnetic vector potential, and $d\mathbf{l} \equiv ds\,\mathbf{e_s} + d\theta\,\mathbf{e}_\theta + d\zeta\,\mathbf{e}_\zeta$ is a line segment along a 'trial' curve, $\mathcal{C}$, which is described by $s(\zeta)$ and $\theta(\zeta)$ with ζ used to describe position along the curve.

2. In the following, it is assumed that the vector potential is given,

$$A_\theta \equiv A_\theta(s,\theta,\zeta) = \sum_j A_{\theta,j}(s)\cos(m_j\theta - n_j\zeta), \tag{2}$$

$$A_\theta \equiv A_\zeta(s,\theta,\zeta) = \sum_j A_{\zeta,j}(s)\cos(m_j\theta - n_j\zeta). \tag{3}$$

3. The computational task is to construct extremizing periodic curves.

1.1.2 discretization of trial curve

1. A practical discretization of $\mathcal{C}$ is given [3]

$$\left.\begin{array}{ccl} s & = & s_i \\ \theta & = & \theta_{i-1} + \dot{\theta}_i\,(\zeta - \zeta_{i-1}) \end{array}\right\} \text{for } \zeta \in [\zeta_{i-1}, \zeta_i], \tag{4}$$

where $\zeta_i \equiv i\,\Delta\zeta$, $\Delta\zeta \equiv \pi/N$ where N is a resolution parameter, and $\dot{\theta}_i \equiv (\theta_i - \theta_{i-1})/\Delta\zeta$.

2. The curve is now described by the s_i and the θ_i.

1.1.3 periodicity constraint

1. The periodicity constraint, $\theta(\zeta + 2\pi\,q) = \theta(\zeta) + 2\pi\,p$, is enforced by the constraint

$$\theta_{2qN} = \theta_0 + 2\pi\,p \tag{5}$$

2. The degrees of freedom in the curve are thus s_i for $i = 1, \ldots, 2qN$ and θ_i for $i = 0, \ldots, 2qN - 1$.

1.1.4 piecewise action integral

1. Using this representation for the trial curve, the action integral becomes

$$S = \sum_{i=1}^{2qN} S_i(\theta_{i-1}, \theta_i, s_i) \tag{6}$$

where

$$\begin{aligned} S_i &\equiv \int_{\zeta_{i-1}}^{\zeta_i} \mathbf{A} \cdot d\mathbf{l} & (7) \\ &= \int_{\zeta_{i-1}}^{\zeta_i} \left(A_\theta \, \dot{\theta}_i + A_\zeta \right) d\zeta & (8) \\ &= a_1 \Delta\zeta + \sum_j a_j \lambda_{j,i}, & (9) \end{aligned}$$

where $a_j \equiv A_{\theta,j}(s_i) \, \dot{\theta}_i \, + A_{\zeta,j}(s_i)$, $\lambda_{j,i} \equiv [\sin(\alpha_{j,i}) - \sin(\alpha_{j,i-1})]/(m_j \dot{\theta}_i - n_j)$, and $\alpha_{j,i} \equiv (m_j \theta_i - n_j \zeta_i)$, and where the summation over j excludes the $(m_j, n_j) = (0, 0)$ component.

1.1.5 conditions for extrema

1. The action integral is extremized when

$$\begin{aligned} \frac{\partial S}{\partial s_i} &= 0 & (10) \\ \frac{\partial S}{\partial \theta_i} &= 0. & (11) \end{aligned}$$

2. Assume that the θ curve is given and the extremizing s curve is to be constructed. We must solve

$$\frac{\partial S}{\partial s_i} = \frac{\partial S_i}{\partial s_i} = a_1' \Delta\zeta + \sum_j a_j' \lambda_{j,i}. \tag{12}$$

3. Define $f(s_i) \equiv a_1' \Delta\zeta + \sum_j a_j' \lambda_{j,i}$. A one-dimensional Newton method can be employed to find $f(s_i + \delta s_i) \approx f(s_i) + f'(s_i) \, \delta s_i = 0$, where $f'(s_i) \equiv a_1'' \Delta\zeta + \sum_j a_j'' \lambda_{j,i}$.

4. Note that the solution, s_i, depends only on θ_{i-1} and θ_i, so that $s_i = s_i(\theta_{i-1}, \theta_i)$. The action integral now becomes a function only of the θ_i.

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